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Can international macroeconomic models explain low-frequency movements of real exchange rates? $\stackrel{\leftrightarrow}{\sim}$



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1. Introduction

This paper challenges the conventional wisdom that a baseline international real business cycle (IRBC) two-country, two-good model cannot generate either enough volatility or enough persistence in the real exchange rate (RER) when compared to the data. When the object of

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ABSTRACT

Real exchange rates exhibit important low-frequency fluctuations. This makes the analysis of real exchange rates at all frequencies a more sound exercise than the typical business cycle one, which compares actual and simulated data after the Hodrick-Prescott filter is applied to both. A simple two-country, two-good, international real business cycle model can explain the volatility of the real exchange rate when all frequencies are studied. The puzzle is that the model generates too much persistence of the real exchange rate instead of too little, as the business cycle analysis asserts. We show that the introduction of input adjustment costs in production, cointegrated productivity shocks across countries, and lower home bias allows us to reconcile theory and this feature of the data.

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interest is RER fluctuations at all frequencies, instead of business cycle (BC) frequencies only, this model can explain the standard deviation of the U.S. dollar RER. However, the model implies a higher persistence of the RER than in the data.

We advocate that analyzing RER fluctuations at all frequencies is a more compelling exercise than just studying the BC ones. Spectral analysis shows that most of the variance of the RER in the data can be assigned to low-frequency movements (about 70%), while movements at BC frequencies account for only a small share of the RER fluctuations (just 25%). The baseline IRBC model accounts for the area below the spectrum of the RER, i.e., its standard deviation, but not for its shape, since it places a larger share of fluctuations of the RER in lowfrequency movements than occurs in the data. We call this shortcoming of the model the "excess persistence of the RER" puzzle. We show that extending the model to consider adjustment costs in the composition of domestic and imported intermediate inputs and lower home bias helps to solve this puzzle (i.e., replicating the shape of the spectrum) while still explaining the standard deviation of the RER (i.e., the area below the spectrum).

Since the seminal works of Backus et al. (1992) and Baxter and Crucini (1995), the IRBC literature has been preoccupied with explaining

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the international transmission of shocks, the cyclical comovement of variables across countries, and the behavior of international relative prices. As in the real business cycle (RBC) literature, the IRBC literature mainly concentrates on explaining the BC fluctuations of the data. The success of the model is measured by its ability to reproduce selected second moments of Hodrick-Prescott (HP) filtered data, which removes trends and low-frequency movements. Other papers use instead the band-pass filter, as described in Baxter and King (1999) or Christiano and Fitzgerald (2003). The researcher compares the second moments of actual data with those implied by the artificial data generated by the model after the same detrending procedure has been applied to both. One of the most relevant facts in the HP-filtered data is that international relative prices are more volatile than output and highly persistent. IRBC models with reasonable calibrations have a hard time reproducing these features. In earlier work Backus et al. (1994) and Stockman and Tesar (1995) showed that IRBC models cannot match the volatility of the (HP-filtered) terms of trade, while, in a more recent contribution, Heathcote and Perri (2002) have pointed out the standard IRBC model's inability to explain the volatility and persistence of the (HP-filtered) RER.

In this paper, we first argue that analyzing only the BC fluctuations of the RER leads researchers to miss a large part of the story. The reason is as follows. The top panel in Fig. 1 plots the (log) U.S. dollar RER along with its implied HP-filtered "trend" using a bandwidth of 1600. Just from eyeballing, it is evident that most of the fluctuations in the U.S. dollar RER have been low-frequency movements. This observation is confirmed by the spectral analysis that we perform in Section 2: most of the variation of the RER in the data is at frequencies lower than BC fluctuations (it is 75% for the U.S. dollar, and between 60 and 76.5% depending on the currency we examine). These low-frequency movements are removed by HP-filtering.¹

Second, motivated by the argument above, we propose to analyze the fluctuations of the RER at all frequencies instead. Therefore, we need to consider a model able to generate low-frequency fluctuations in the RER. Our baseline model is an extension of the two-country, two-good model of Heathcote and Perri (2002) in which stochastic processes for total factor productivity (TFP) are non-stationary but cointegrated across countries.² We show that the model can explain about 80% of the standard deviation of the RER in the data while closely matching the volatility of output growth when we use a benchmark calibration of the model, including a value of 0.85 for the elasticity of substitution between intermediate inputs in the production of the final good. However, in the model, the RER is too persistent and the spectrum places too much weight on low-frequency fluctuations (in the model 85% of the variance is caused by low-frequency fluctuations while it is 70% in the data). In order to solve this shortcoming, we extend the model with adjustment costs in the use of intermediate imported inputs for the production of the final good (see Erceg et al. (2006)). The presence of these costs allows us to combine a low short-run elasticity of substitution between imported and domestic intermediate goods, which is needed to increase the volatility of the RER at BC frequencies, with a higher long-run elasticity, which is needed to reduce the excessive volatility of the RER at low frequencies. We show how these input adjustment costs, together with lower home bias, help to solve the puzzle by increasing the impact response of the RER in the short run while reducing it at long-run horizons in the model. The calibration of a lower home bias is consistent with recent data that show more trade openness for the U.S.



Fig. 1. Log RER, autocorrelation function, and spectral density of the U.S. dollar.

Moreover, our model can explain an important fact in international macroeconomics at several frequencies. Since the seminal paper by Backus and Smith (1993), the literature has been preoccupied with matching the correlation between the ratio of the relative consumption of two countries and the RER at BC frequencies. This correlation tends to be close to one in the standard model, even under cointegrated shocks, while it is negative in the data. Corsetti et al. (2008a) were the first ones to propose a solution to this puzzle under different specifications of international asset markets, elasticities of substitution between types of goods, and persistence of the underlying productivity shocks. However, their analysis focused on HP-filtered data. Recently, an empirical paper by Corsetti et al. (2012) has confirmed the Backus and Smith (1993) results at low, BC and high frequencies for a large sample of countries. The extended model in this paper is in fact able to explain the negative correlation at all frequencies. However, it should be noted that this mechanism is at odds with existing VAR evidence, as presented in Corsetti et al. (2014).

The paper is organized as follows: Section 2 presents the spectral analysis of the U.S. dollar RER as well as that of other main currencies. Section 3 discusses the related literature, while Section 4 presents a baseline IRBC model. Section 5 presents the calibration and the results of the baseline model. In Section 6, we present the extensions to the model and show how they help reconcile theory and evidence. Section 7 concludes.

2. Spectral analysis of the RER

In this section we study the spectrum of the RER of six main currencies: the U.S. dollar, the euro, the U.K. pound sterling, the Japanese yen, and the Canadian and Australian dollars. In order to find the longest possible time series for each currency, we choose between the IMF's International Financial Statistics (IFS) database, the measure constructed from national central banks, or other measures. We verify that for the period during which both measures overlap the correlation is very high, denoting that all sources use similar methodologies to construct the RER series.

The sample period is 1973Q1–2013Q3 unless otherwise noted. Our data sources are as follows: for the U.S. dollar we obtain the real effective exchange rate (REER) series from the Federal Reserve's Real Broad

¹ The RER in emerging markets can have a trend, in particular in those emerging economies that experience higher productivity growth rates than advanced economies. In that case, the use of a trend/cycle decomposition would be justified. However, most of the IRBC literature focuses on explaining the RER of the U.S. dollar vis-a-vis the currencies of other industrialized countries. In that case RERs are a highly persistent series, but they do not have a trend.

² In a related work, Rabanal et al. (2011) show that cointegrated TFP shocks improve the model's ability to explain certain features of the HP-filtered data, including RER volatility.

Trade-Weighted Value of the U.S. dollar. For the euro area, we use the Broad REER series computed by J.P. Morgan. For the Canadian dollar and the U.K. pound sterling we use the IFS measure (sample period 1975Q2–2013Q3). For the Australian dollar, we use the REER measure constructed by the Reserve Bank of Australia. For the Japanese yen, we use the REER measure constructed by the Bank of Japan using the BIS methodology.

The spectrum contains the same information as auto-correlations and it allows us to decompose the variance of the RER across different frequencies. In order to estimate the spectrum we use the modified Bartlett kernel methodology described in Section 6.4 of Hamilton (1994). In Fig. 1 we present the time series for the (log) U.S. dollar RER along with its implied HP-filtered "trend," its autocorrelogram, and the estimated spectrum density. From the first two panels in Fig. 1, we can observe that the U.S. dollar RER does not have an evident time trend. At the same time, it is a highly persistent series: the autocorrelogram decays monotonically as the lag length is increased, but it decays slowly. As a result, the correlation between the RER and its own 15th lag is basically zero. In the bottom panel of Fig. 1 we present the estimated spectrum, where we have shaded the area corresponding to BC frequencies: most fluctuations occur at low frequencies. The facts presented in Fig. 1 are common to all of the other major currencies we studied.³ In all cases, the low-frequency movements implied by the HP-filtered "trend" are guite sizable, the autocorrelogram decays at a slow rate (but fast enough to suggest that there is not a unit root), and the estimated spectrum suggests that most fluctuations occur at low frequencies.

We put some numbers to this last claim by decomposing the variance of each RER into BC frequencies (8 to 32 quarters), lower than BC frequencies (more than 32 quarters) and higher than BC frequencies (less than 8 quarters) in Table 1. We also report the results coming from constructing our own U.S. dollar RER series by recomputing the RER against the following four countries: Japan, Canada, the U.K., and Australia, and the euro area. These four countries and the euro area are used later in the paper to calibrate the "rest of the world" TFP process; hence, for consistency it makes sense to compute the RER vis-a-vis this group. We compute bilateral RERs and aggregate them by using the currency weights from the Broad Index of the Foreign Exchange Value of the dollar computed by the U.S. Federal Reserve.⁴

As shown in Table 1, most of the variance of the U.S. dollar RER (75.1%) is concentrated at low frequencies (less than 32 quarters), while 20% of the variance is attributed to BC frequencies and only 4.9% occurs at high frequencies. Our measure vis-à-vis the main industrialized countries behaves similarly. Taking an international comparison, the fraction of the variance concentrated at low-frequency movements ranges from 59.9% for the U.K. pound sterling to 76.5% for the Australian dollar. Therefore, the literature that tries to explain BC-frequency fluctuations of RERs misses a large part of the picture that resides in the lowfrequency end of the spectrum. The finding that most of the variance of the RER is concentrated at low frequencies can be related to two welldocumented facts: first, the large half-life of estimated IRFs of the RER (Rogoff, 1996; Murray and Papell, 2002; and Steinsson, 2008) and second, its hump-shaped dynamics (Huizinga, 1987; Eichenbaum and Evans, 1995; Cheung and Lai, 2000; and Steinsson, 2008). Both the large half-life and the dynamic non-monotonic response pattern are closely related to the high persistence of RERs in the data and to the importance of low-frequency fluctuations.

3. Relationship to the literature

This paper bridges the gap between empirical models and dynamic stochastic general equilibrium (DSGE) models in explaining RER

Table 1

Variance decomposition of the RER (in percent).

| | Low | BC | High |
|----------------------|------|------|------|
| U.S.—Federal Reserve | 75.1 | 20.0 | 4.9 |
| U.S.—Our measure | 72.2 | 22.4 | 5.3 |
| Euro area | 64.3 | 27.7 | 7.8 |
| U.K. | 59.9 | 32.8 | 7.2 |
| Japan | 68.4 | 25.7 | 5.8 |
| Australia | 76.5 | 18.5 | 5.0 |
| Canada | 74.7 | 19.4 | 5.9 |

fluctuations. The empirical literature since the seminal work of Meese and Rogoff (1983) has mostly used univariate and multivariate time series methods to model exchange rates (nominal or real). This analysis is mostly performed at all frequencies. In a recent paper, Steinsson (2008) follows a large literature that models the linear univariate empirical properties of the RER. Other nonlinear univariate time series approaches are reviewed in Sarno (2003). In the multivariate setup, Clarida and Galí (1994) and Faust and Rogers (2003), among many others, have used VAR models to explain the response of exchange rates (both real and nominal) to several shocks. Another branch of the literature studies the role of world and country-specific factors in explaining the comovement of the main macroeconomic variables across countries within the context of dynamic factor models (see, for instance, Mumtaz and Surico (2009)). Other authors examine the relationship between exchange rates (both real and nominal) and fundamentals derived from open economy macro models, such as Engel and West (2005), and Cheung et al. (2005). Finally, Engel and Hamilton (1990) explain long swings in the U.S. dollar RER by estimating a switching regime model with segmented trends.

However, most calibrated DSGE models are typically concerned with explaining the BC fluctuations of the RER and hence analyze HP-filtered data. Since Heathcote and Perri (2002), the literature has been energetically trying to reconcile the discrepancy between theory and HPfiltered RER data, with some success. For example, Chari et al. (2002) show that a monetary economy with monopolistic competition and sticky prices can explain HP-filtered RER volatility if a high degree of risk aversion is assumed. Corsetti et al. (2008a) show that introducing a low elasticity of substitution between types of goods (or a high elasticity together with highly persistent productivity shocks) also helps reconcile theory with the data, and were the first ones to explain the correlation between the ratio of the relative consumption of two countries and the RER at BC frequencies. Rabanal et al. (2011) show that introducing cointegrated TFP processes across countries helps to explain the volatility of the HP-filtered RER. Although such models do a better job explaining the volatility of the HP-filtered RER, they still cannot match its persistence. A number of related papers have tried to tackle the lack of persistence of RER in the model in the context of monetary models (for example, see Bergin and Feenstra (2001); Benigno (2004), or Bouakez (2005)) without completely addressing it.

In this paper we combine the two approaches by comparing the properties of the RER in the DSGE model and in the data, without applying any filtering method. It is also worth noting that a few recent exceptions to this filtering practice arise in the literature that estimates open economy DSGE models with Bayesian methods. Adolfson et al. (2007) and Rabanal and Tuesta (2010) include the log of the RER in the set of observable variables, while Nason and Rogers (2008) use the log of the nominal exchange rate between the U.S. and Canadian dollars in their estimated model. Also, there are some recent exceptions to the practice of focusing only on BC fluctuations of the data and comparing them to the model. Baxter (2011) finds that there is evidence in favor of risk sharing across countries at medium and low frequencies. Corsetti et al. (2012) study the correlation between the RER and the ratio of consumption levels across countries (which is known as the "Backus-Smith puzzle") at both BC and low frequencies. Comin and Gertler (2006) use a medium-scale closed economy model to explain

³ To save space, we do not repeat Fig. 1 for the rest of the major currencies, but they are available upon request.

⁴ For a description see http://www.federalreserve.gov/releases/H10/Weights/.

medium-term fluctuations (between zero and 50 quarters) of the main macroeconomic aggregates of the U.S. economy.

4. The baseline model

As a baseline we use a two-country, two-good model similar to the one described in Backus et al. (1994) and Heathcote and Perri (2002) with a main important difference: (the log of) TFP processes are assumed to be non-stationary but cointegrated across countries. In other words, they follow a VECM process.⁵

To keep exposition to a minimum, we present only the problem of home-country households, home-country firms, and market clearing. Then we will describe the equilibrium conditions. In terms of notation, we use an asterisk superscript when we refer to the foreign-country variable analogous to a home-country variable (i.e., if C_t is consumption in the home country, then C_t^* is consumption in the foreign country). In each country, a single final good is produced by a representative competitive firm that uses intermediate goods from both countries in the production process. These intermediate goods are imperfect substitutes for each other and can be purchased from representative competitive producers of intermediate goods in both countries. Intermediate goods producers use domestic capital and domestic labor in the production process and face a domestic TFP shock. The final good can only be domestically consumed or domestically invested in by domestic households. Thus, all trade of goods between countries occurs at the intermediate goods level. In addition, households trade across countries an uncontingent international riskless bond denominated in units of the home-country intermediate good. No other financial asset is available.

4.1. Households

The representative household of the home country solves:

$$\max_{\{C_t, L_t, X_t, K_t, D_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\tau} (1-L_t)^{1-\tau}\right]^{1-\sigma}}{1-\sigma}$$

subject to the following budget constraint:

$$P_t(C_t + X_t) + P_{H,t}\overline{Q_t}D_t \le P_t(W_tL_t + R_tK_{t-1}) + P_{H,t}[D_{t-1} - \Phi(D_t, A_{t-1})]$$

and the law of motion for capital:

 $K_t = (1 - \delta)K_{t-1} + X_t.$

The following notation is used: β is the discount factor, L_t is the fraction of time allocated to work in the home country, C_t are units of consumption of the final good, X_t are units of investment, and K_t is the capital stock in the home country at the beginning of period t + 1. P_t is the price of the home country final good, which will be defined below; W_t is the hourly wage in the home country, and R_t is the home country rental rate of capital, where the prices of both factor inputs are measured in units of the final good. $P_{H,t}$ is the price of the home-country intermediate good, D_t denotes the holdings of the internationally traded riskless bond that pays one unit of the home-country intermediate t = 1 regardless of the state of nature, and $\overline{Q_t}$ is its price, measured in units of the home-country for $\Phi(\cdot)$ measures the home-country intermediate good. The function $\Phi(\cdot)$ measures the home-country intermediate good.

cost of holding bonds measured in units of the home-country intermediate good. $^{\rm 6}$

Following the existing literature, $\Phi\left(\cdot\right)$ takes the functional form:

$$\Phi(D_t, A_{t-1}) = \frac{\phi}{2} A_{t-1} \left(\frac{D_t}{A_{t-1}} \right)^2$$

where we have modified this function to include the home-country TFP level, A_{tr} which is characterized below, to ensure balanced growth.

4.2. Firms

We now describe the production function and profit maximization problems of the final and intermediate goods producers. Then, we portray technology.

4.2.1. Final goods producers

The final good in the home country, Y_{t} , is produced using homecountry intermediate goods, $Y_{H,t}$, and foreign-country intermediate goods, $Y_{F,t}$, with the following technology:

$$Y_{t} = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$
(1)

where ω denotes the fraction of home-country intermediate goods that are used for the production of the home-country final good and θ is the elasticity of substitution between home-country and foreign-country intermediate goods. Therefore, the representative final good producer in the home country solves the following problem:

$$\max_{Y_{t},Y_{H,t},Y_{F,t}} P_{t}Y_{t} - P_{H,t}Y_{H,t} - P_{F,t}^{*}Y_{F,t}$$

subject to the production function (1), where $P_{F,t}^*$ is the price of the foreign-country intermediate good in the home country.

4.2.2. Intermediate goods producers

The representative intermediate goods producer in the home country uses domestic labor and domestic capital in order to produce homecountry intermediate goods and sells her product to both the homecountry and foreign-country final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits by solving:

$$\max_{L_t,K_{t-1}} P_{H,t} Y_{H,t} + P_{H,t}^* Y_{H,t}^* - P_t (W_t L_t + R_t K_{t-1})$$

subject to the production function:

$$Y_{H,t} + Y_{H,t}^* = A_t^{1-\alpha} K_{t-1}^{\alpha} L_t^{1-\alpha}$$
(2)

where $Y_{H,t}^*$ is the amount of home-country intermediate goods sold to the foreign-country final good producers and $P_{H,t}^*$ is the price of the home-country intermediate good in the foreign country.

4.2.3. TFP processes

We assume that $\log A_t$ and $\log A_t^*$ are cointegrated of order C(1, 1). This assumption involves specifying the following VECM for the law of motion driving the log first difference of TFP processes for both the home and the foreign country:

$$\begin{pmatrix} \Delta \log A_t \\ \Delta \log A_t^* \end{pmatrix} = \begin{pmatrix} c \\ c^* \end{pmatrix} + \begin{pmatrix} \kappa \\ \kappa^* \end{pmatrix} \left[\log A_{t-1} - \gamma \log A_{t-1}^* - \log \xi \right] + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix} (3)$$

⁵ Rabanal et al. (2011) show that TFP processes between the U.S. and a sample of the main industrialized countries are cointegrated and that the low estimated speed of convergence to the cointegrating relationship is a key ingredient for the model to explain the volatility of the RER at BC frequencies. Here, we examine how the same model performs in explaining movements of the RER at all frequencies. Since the model is the same as in the above-mentioned reference, we just show the main functional forms and optimality conditions and refer the reader to the original paper for a detailed derivation.

⁶ The $\Phi(\cdot)$ cost is introduced to ensure stationarity of the level of D_t in IRBC models with incomplete markets, as discussed by Heathcote and Perri (2002). In this baseline model we choose the cost to be numerically small, so it does not affect the dynamics of the rest of the variables. This will not be the case when we analyze some of the extensions.

where $(1, -\gamma)$ is the cointegrating vector, ξ is the constant in the cointegrating relationship, $\varepsilon_t \sim N(0, \sigma)$ and $\varepsilon_t^* \sim N(0, \sigma^*)$, ε_t and ε_t^* can be correlated, and Δ is the first-difference operator.

4.3. Market clearing

The model is closed with the following market clearing conditions in the final good markets:

$$C_t + X_t = Y_t \tag{4}$$

and in the international bond market:

$$D_t + D_t^* = 0. (5)$$

4.4. Equilibrium conditions

At this point, it is useful to define the following relative prices: $\tilde{P}_{H,t} = \frac{P_{H,t}}{P_t}$, $\tilde{P}_{F,t}^* = \frac{P_{F,t}^*}{P_t^*}$ and $RER_t = \frac{P_t^*}{P_t}$ where P_t^* is the price of the foreign-country final good. Note that $\tilde{P}_{H,t}$ is the price of home-country intermediate goods in terms of the home-country final good, $\tilde{P}_{F,t}^*$ is the price of foreign-country intermediate goods in terms of the home-country final good, $\tilde{P}_{F,t}^*$ is the price of foreign-country intermediate goods in terms of the foreign-country final good, which appears in the foreign-country's budget constraint, and *RER*_t is the RER between the home and foreign countries. The law of one price (LOP) holds: $P_{H,t} = P_{H,t}^*$ and $P_{F,t} = P_{F,t}^*$.

The equilibrium conditions include the first-order conditions of households, and intermediate and final goods producers in both countries, as well as the relevant laws of motion, production functions, and market clearing conditions. Here, we detail the home-country equilibrium conditions only. The foreign-country conditions are very similar, with the appropriate change of notation. The marginal utility of consumption and the labor supply are given by:

 $U_{C_t} = \lambda_t,$

 $\frac{U_{L_t}}{U_{C_t}} = W_t,$

where U_x denotes the partial derivative of the utility function U with respect to variable x. The first-order condition with respect to capital delivers an intertemporal condition that relates the marginal rate of consumption to the rental rate of capital and the depreciation rate:

$$\lambda_t = \beta E_t [\lambda_{t+1} (R_{t+1} + 1 - \delta)].$$

The law of motion of capital is:

 $K_t = (1 - \delta)K_{t-1} + X_t.$

The optimal savings choice delivers the following expression for the price of the riskless bond:

$$\overline{Q_t} = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{\widetilde{P}_{H,t+1}}{\widetilde{P}_{H,t}} \right) - \frac{\Phi'(D_t)}{\beta}$$

The next condition uses the expression for the price of the bond in both countries to derive the expression for optimal risk sharing across countries:

$$E_t \left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{\widetilde{P}_{H,t+1}}{\widetilde{P}_{H_t}} \frac{RER_t}{RER_{t+1}} - \frac{\lambda_{t+1}}{\lambda_t} \frac{\widetilde{P}_{H,t+1}}{\widetilde{P}_{H,t}} \right] = -\frac{\Phi'(D_t)}{\beta}$$

From the intermediate goods producers' maximization problems, labor and capital are paid their marginal product, where the rental rate of capital and the real wage are expressed in terms of the final good in each country:

$$W_t = (1 - \alpha) \widetilde{P}_{H,t} A_t^{1 - \alpha} K_{t-1}^{\alpha} L_t^{-\alpha}$$

and

 $R_t = \alpha \widetilde{P}_{H,t} A_t^{1-\alpha} K_{t-1}^{\alpha-1} L_t^{1-\alpha}.$

From the final good producers' maximization problem, the demand for home and foreign country intermediate goods depends on their relative price:

$$Y_{H,t} = \omega \widetilde{P}_{H,t}^{-\theta} Y_t, \tag{6}$$

$$Y_{F,t} = (1 - \omega) \left(\tilde{P}_{F,t}^* RER_t \right)^{-\theta} Y_t.$$
⁽⁷⁾

Using the production functions of the final good:

$$Y_t = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

eqs. (6) and (7), the final good deflator in the home-country is:

$$P_{t} = \left[\omega P_{H,t}^{1-\theta} + (1-\omega) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Hence, given that the LOP holds, the RER is equal to:

$$RER_{t} = \frac{P_{t}^{*}}{P_{t}} = \frac{\left[\omega P_{F,t}^{1-\theta} + (1-\omega)P_{H,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[\omega P_{H,t}^{1-\theta} + (1-\omega)P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}}.$$

Note that the only source of RER fluctuations is the presence of home bias ($\omega > 1/2$). Also, intermediate goods, final good, and bond markets clear as in Eqs. (2), (4), and (5). Finally, the law of motion of the level of bonds:

$$\widetilde{P}_{H,t}\overline{Q}_{t}D_{t} = \widetilde{P}_{H,t}Y_{H,t}^{*} - RER_{t}\widetilde{P}_{F,t}^{*}Y_{F,t} + \widetilde{P}_{H,t}D_{t-1} - \widetilde{P}_{H,t}\Phi(D_{t},A_{t-1})$$
(8)

is obtained using the household budget constraint and the fact that intermediate and final good producers make zero profits. Finally, the TFP shocks follow the VECMs described above. Since the model is nonstationary, we need to normalize it and check for the existence of a balanced growth path. Rabanal et al. (2011) find that the estimated γ is one, which is a sufficient condition for balanced growth to exist in this economy (in addition to the standard restrictions on technology and preferences, as in King et al. (1988)). Hence, along the balanced growth path, real variables in each country grow at the same rate as its TFP. To solve and simulate the model, we normalize real variables in each country by the lagged level of TFP in that country to obtain a stationary system. Then, we take a log-linear approximation to the normalized equilibrium conditions.

5. Results of the baseline model

In this section we describe the results of the baseline model. First, we describe the benchmark calibration for the baseline model. Then, we show that the baseline model with the benchmark calibration can closely replicate the standard deviation of the RER when all frequencies are considered. In other words, it reproduces the area below the RER spectrum. However, we also show that the model cannot replicate the shape of the spectrum. It assigns too much variance of the RER to fluctuations with frequencies below BC ones when compared to the data. This is what we call the "excess persistence of the RER" puzzle. Finally, we show that these findings are robust to some standard changes in the

literature such as assuming stationary TFP shocks or cointegrated investment-specific technology (IST) shocks.

5.1. Benchmark calibration for the baseline model

Our benchmark calibration closely follows that in Heathcote and Perri (2002), to allow a proper comparison. The model is quarterly. The discount factor β is set equal to 0.99, which implies an annual real rate of 4%. In the utility function, we set the consumption share τ to 0.34 and the coefficient of risk aversion σ to 2. Parameters on technology are fairly standard in the literature. Thus, the depreciation rate δ is set to 0.025 the capital share of output α is set to 0.36, and the ratio of intermediate inputs in the production of the final good ω is set to 0.9, which matches the actual import/output ratio in the steady state.⁷ We calibrate the elasticity of substitution between intermediate goods to $\theta = 0.85$. We will also consider other values of θ to check the robustness of our results. We assume a cost of bond holdings, ϕ , of 1 basis point (0.01).

The calibration of the VECM process follows the estimates in Rabanal et al. (2011). Their paper constructed a series of TFP for the United States and another series for a "rest of the world" aggregate of the main industrialized trade partners of the U.S. (Australia, Canada, Euro Area, Japan, and the U.K.) using data on output, employment, hours and capital stock. They tested for and confirmed the presence of unit roots in each series and cointegration between the two TFP series using Johansen's (1991) test. Finally, they estimate a process like Eq. (3). In addition to not rejecting that $\gamma = 1$, they find that (i) zero lags are necessary and (ii) they cannot reject that $\kappa = -\kappa^*$ (i.e., that the speed of convergence to the cointegrating relationship is the same for both countries). Following their estimates, we set $\gamma = 1$, $\kappa = -0.007$, c = 0.001, $c^* = 0.006$, $\sigma = 0.0108$ and $\sigma^* = 0.0088$.

5.2. Matching the RER spectrum

Fig. 2 presents the spectrum of the RER implied by our baseline model under the benchmark calibration and compares it with the estimated spectrum for our constructed measure of the U.S. dollar RER. Our measure includes the same countries we considered when constructing the "rest of the world" TFP. Since we can compute the theoretical moments of the growth rates of variables and of the RER implied by the model, it is possible to compute the theoretical spectrum of the RER.

Table 2 displays some key statistics of the RER implied by the baseline model under the benchmark calibration and compares them to the data. The same table also shows results for alternative values for θ . The baseline model with the benchmark calibration can closely replicate the standard deviation of the RER when compared to the data (8.33 in the model versus 10.91 in the data), and also gets the standard deviation of output growth about right (0.75 in the model versus 0.81 in the data). However, Fig. 2 and Table 2 highlight the model's main problem. It assigns too large of a share of the variance of the RER to low-frequency fluctuations: almost 89% in the model versus 72.2% in the data. This result is related to the usual finding that the model cannot explain the volatility of the HP-filtered RER because it is precisely the low-frequency component that is removed with the HP filter.⁸ As mentioned above, we call this discrepancy between the model and the data the "excess persistence of the RER" puzzle.

Next, we present results for $\theta = 0.62$. This is a relevant value because Rabanal et al. (2011) found that it allowed the model to match the relative volatility of the HP-filtered RER with respect to HP-filtered output. The model now implies a larger standard deviation of the RER than in the data (16.2 versus 10.91). The shape of the RER spectrum does not



Fig. 2. Comparison between the model and the data.

change much and most of the volatility (88%) is again assigned to low-frequency movements. Hence, in order to match the standard deviation of the HP-filtered RER, the model generates too much volatility of the RER at all frequencies. Finally, we also analyze the implications of the value of $\theta = 1.5$ (which is used by Chari et al. (2002), and Erceg et al. (2006)). As expected, the model explains less of the volatility of the RER (3.55 versus 10.91) and the shape of the spectrum is basically the same. Hence, while the standard deviation of the RER at all frequencies is inversely related to the elasticity of substitution, θ , the shape of the spectrum seems to be invariant to it. Low values of θ help to explain RER variance (the area under the spectrum) but do not solve the "excess persistence of the RER" puzzle (the shape of the spectrum).

5.3. Some robustness

We have found that the model's main failure is the "excess persistence of the RER" puzzle. In this subsection, we perform some robustness analysis to determine whether the puzzle survives after simple modifications of the model. In particular, we analyze two variations that involve different assumptions on the shocks that drive the model. First, we use the Heathcote and Perri (2002) estimates for the joint evolution of stationary TFP shocks. Second, we use the cointegrated TFP and IST shocks as in Mandelman et al. (2011). The results are reported in Table 3. We use the label "Stationary" to refer to the Heathcote and Perri (2002) model, and we use "TFP and IST" to refer to the model with cointegrated TFP and IST shocks.

Heathcote and Perri (2002) estimate a VAR(1) in levels to model the joint behavior of TFP processes across countries (the U.S. and a "rest of the world" aggregate). When we use their estimated process, we find that their model cannot explain the volatility of the RER.

| Table 2 | |
|------------------------------------------|--|
| Implications of the model with only TFP. | |

| | Standard deviation | | Frequen | | |
|-----------------|--------------------|---------------|---------|------|------|
| | RER | Output growth | Low | BC | High |
| U.S. data | 10.91 | 0.81 | 72.2 | 22.4 | 5.3 |
| $\theta = 0.85$ | 8.33 | 0.75 | 88.4 | 8.8 | 2.8 |
| $\theta = 062$ | 16.02 | 0.68 | 88.2 | 8.9 | 2.9 |
| $\theta = 1.5$ | 3.55 | 0.85 | 89.0 | 8.4 | 2.6 |

Note: RER denotes the log of the RER. Output is real GDP.

Growth rates are computed taking the first differences of the logs.

 $^{^7\,}$ In Section 6, we discuss how a lower home bias parameter (ω) is needed to obtain a better fit to the data.

⁸ Rabanal et al. (2011) found that when $\theta = 0.85$, this exact same model can explain only about half of the volatility of the HP-filtered RER.

Table 3 Robustness.

| | Standar | d deviation | Frequency of RER | | | |
|------------------------------|---------|---------------|------------------|------|------|--|
| | RER | Output growth | Low | BC | High | |
| U.S. data | 10.91 | 0.81 | 72.2 | 22.4 | 5.3 | |
| Stationary, $\theta = 0.85$ | 4.03 | 0.92 | 85.3 | 11.3 | 3.3 | |
| Stationary, $\theta = 062$ | 7.35 | 0.87 | 85.5 | 11.1 | 3.4 | |
| Stationary, $\theta = 1.5$ | 1.86 | 0.97 | 85.8 | 11.3 | 2.9 | |
| TFP and IST, $\theta = 0.85$ | 8.58 | 0.76 | 88.5 | 8.7 | 2.8 | |
| TFP and IST, $\theta = 062$ | 16.61 | 0.65 | 88.2 | 8.9 | 2.9 | |
| TFP and IST, $\theta = 1.5$ | 3.77 | 0.82 | 89.1 | 8.3 | 2.6 | |

Note: See note in Table 2.

With their benchmark calibration using $\theta = 0.85$ the model explains less than 40% of the standard deviation of the RER. Even reducing the value of θ to 0.62 is not enough. As explained in Rabanal et al. (2011), the presence of a common unit root and slow transmission of shocks across countries is a crucial ingredient for explaining large RER volatility, and this feature is missing in Heathcote and Perri (2002). Note that the model with stationary TFP shocks assigns somewhat less volatility to low-frequency fluctuations than the baseline model, but the differences are not relevant and the results are still far away from matching the data. Next, we look at what happens when we go back to the case of cointegrated TFP shocks but also introduce cointegrated IST shocks, as estimated by Mandelman et al. (2011). Including IST shocks results in marginal changes for explaining RER volatility and the spectrum.

The conclusion of this section is that, while the baseline model can replicate the area below the spectrum of the RER for low values of the elasticity of substitution, it has a hard time reproducing its shape because too much weight is placed on low-frequency fluctuations. In addition, none of the modifications analyzed, which involve only different assumptions on the exogenous shocks driving the model, help in solving the puzzle. In the next section, we modify the model so that it can replicate not only the area below the RER spectrum (the standard deviation) but also its shape (the persistence), i.e., we introduce an extended model that can solve the puzzle.

6. Extensions to the baseline model

In this section, we will add two ingredients to the baseline model that will help us solve the puzzle while still replicating the variance of RER. First, we consider adjustment costs in the use of intermediate imported inputs for the production of the final good, and second, we analyze the role of lower home bias.

6.1. Adjustment costs in the use of intermediate imported inputs

The first additional ingredient will be to assume adjustment costs in the use of intermediate imported inputs for the production of the final good. As we will see below, this feature will allow us to combine low short-run elasticities of substitution between intermediate goods with high long-run ones. The empirical literature that estimates trade elasticities argues that, due to the slow adjustment of quantities in response to prices, elasticities of substitution differ in the short run and in the long run. For instance, Hooper et al. (2000) estimate import and export equations for the G-7 countries and show that the long-run elasticities are much higher than the short-run ones.

In order to include input adjustment costs, we follow Erceg et al. (2006). Hence, the production function is now:

$$Y_t = \left[\omega^{\frac{1}{\theta}} Y_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (\varphi_t Y_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

As we will see below, θ is now the elasticity of substitution between home-country and foreign-country intermediate goods in the long run. The input adjustment, φ_{t} follows the following functional form:

$$\varphi_t = \left[1 - \frac{\iota}{2} \left(\frac{Y_{F,t} / Y_{H,t}}{Y_{F,t-1} / Y_{H,t-1}} - 1 \right)^2 \right].$$
(9)

With this specification, changing the ratio of home-country to foreign-country intermediate goods reduces the efficiency of the imported intermediate input.⁹ There are no direct available estimates of the cost function (9). Hence, how can we interpret the ι parameter and the cost function? Suppose that the ratio $\frac{Y_{F,t-1}/Y_{H,t}}{Y_{F,t-1}/Y_{H,t-1}}$ deviates by 1% from its steady-state value at time *t*. Then the value of $\varphi_t = 1 - \frac{\iota}{2} (0.01)^2$. With a value of $\iota = 200$, then $\varphi_t = 0.99$ and given an $\omega = 0.9$ home-country output will be 0.1% smaller than without the presence of this cost.

The input adjustment cost function depends on variables dated at t-1, and hence this introduces an intertemporal dimension to the final good producers' profit maximization problem. We use the domestic households' stochastic discount factor to discount future profits. The representative final good producer in the home country solves the following problem:

$$\max_{Y_{t+k}, Y_{H,t+k}, Y_{F, t+k}} E_{t} \sum_{k=0}^{\infty} \beta^{k} \Lambda_{t+k} \Big(P_{t+k} Y_{t+k} - P_{H,t+k} Y_{H,t+k} - P_{F,t+k} Y_{F,t+k} \Big)$$

subject to the production function (1) and the input adjustment cost function (9). Note that $\beta^k \Lambda_{t+k} = \beta^k (\lambda_{t+k}/P_{t+k})/(\lambda_t/P_t)$ is the stochastic discount factor. The first-order conditions of the problem are given by:

$$P_t \frac{\partial Y_t}{\partial Y_{H,t}} + \beta E_t \left(\Lambda_{t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{H,t}} \right) = P_{H,t}$$

and

$$P_{t} \frac{\partial Y_{t}}{\partial Y_{F,t}} + \beta E_{t} \left(\Lambda_{t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{F,t}} \right) = P_{F,t}$$

Using the previous functional forms we obtain the following expressions:

$$\frac{P_{H,t}}{P_{t}} = Y_{t}^{\frac{1}{2}} \left[\omega^{\frac{1}{2}} Y_{H,t}^{-\frac{1}{2}} + (1-\omega)^{\frac{1}{2}} \left(Y_{F,t} \right)^{\frac{\theta-1}{\theta}} (\varphi_{t})^{\frac{-1}{\theta}} \left(\frac{Y_{F,t}/Y_{H,t}}{Y_{F,t-1}/Y_{H,t-1}} - 1 \right) \frac{Y_{F,t}/\left(Y_{H,t}\right)^{2}}{Y_{F,t-1}/Y_{H,t-1}} \right] -\beta E_{t} \left\{ \left(\frac{\lambda_{t+1}}{\lambda_{t}} \right) Y_{t+1}^{\frac{1}{\theta}} \left[(1-\omega)^{\frac{1}{\theta}} \left(Y_{F,t+1} \right)^{\frac{\theta-1}{\theta}} (\varphi_{t+1})^{\frac{-1}{\theta}} \left(\frac{Y_{F,t+1}/Y_{H,t+1}}{Y_{F,t}/Y_{H,t}} - 1 \right) \frac{Y_{F,t+1}/Y_{H,t+1}}{Y_{F,t}} \right] \right\}$$
(10)

and

$$\frac{P_{F,t}}{P_{t}} = Y_{t}^{\frac{1}{\theta}} \left[(1 - \omega)^{\frac{1}{\theta}} \left(\varphi_{t} Y_{F,t} \right)^{\frac{-1}{\theta}} \left(\varphi_{t} - \iota Y_{F,t} \left(\frac{Y_{F,t} / Y_{H,t}}{Y_{F,t-1} / Y_{H,t-1}} - 1 \right) \frac{1 / Y_{H,t}}{Y_{F,t-1} / Y_{H,t-1}} \right) \right] \\
+ \beta P_{t} E_{t} \left\{ \left(\frac{\lambda_{t+1}}{\lambda_{t}} \right) Y_{t+1}^{\frac{1}{\theta}} \left\{ (1 - \omega)^{\frac{1}{\theta}} \left(\varphi_{t+1} Y_{F,t+1} \right)^{\frac{-1}{\theta}} \right. \tag{11} \\
\times \left[Y_{F,t+1} \iota \left(\frac{Y_{F,t+1} / Y_{H,t+1}}{Y_{F,t} / Y_{H,t}} - 1 \right) \frac{Y_{F,t+1} / Y_{H,t+1}}{\left(Y_{F,t} \right)^{2} / Y_{H,t}} \right] \right\} \right\}.$$

⁹ Obstfeld and Rogoff (2000) analyze the role of transportation costs (in the form of iceberg costs) in explaining several puzzles of international macroeconomics. However, they conclude that this type of friction alone cannot solve the puzzle of the high volatility of RERs, which they label "the exchange rate disconnect puzzle."

| Table 4 | |
|--------------------------------|------|
| The role of input adjustment c | osts |

| | Standard deviation | | Frequency of RER | | | |
|---------------|--------------------|---------------|------------------|------|------|--|
| | RER | Output growth | Low | BC | High | |
| U.S. data | 10.91 | 0.81 | 72.2 | 22.4 | 5.3 | |
| $\iota = 0$ | 1.58 | 0.93 | 89.1 | 8.4 | 2.5 | |
| $\iota = 125$ | 3.6 | 0.87 | 84.8 | 11.7 | 3.5 | |
| $\iota = 250$ | 6.2 | 0.83 | 83.1 | 13.0 | 3.9 | |
| $\iota = 375$ | 9.5 | 0.78 | 82.5 | 13.4 | 4.1 | |
| $\iota = 500$ | 13.92 | 0.72 | 82.2 | 13.6 | 4.2 | |

Note: See note in Table 2.

Foreign-country intermediate goods producers face the same problem, which we do not describe because of space considerations. We calibrate the parameters as described in Section 1 except the long-run elasticity of substitution between intermediate goods is now set to a value of 3. This value is higher than that typically used in open economy macro models (Chari et al. (2002), and Erceg et al. (2006) use $\theta = 1.5$), but consistent with micro-level estimates (see, for instance, Imbs and Méjean (2009)). We now vary the degree of the cost, ι , and look at the implications for the model. The results are reported in Table 4.

Introducing an input adjustment cost has important implications for the RER.¹⁰ As expected with a high elasticity of substitution of $\theta = 3$, when $\iota = 0$ the model does not generate enough volatility of the RER and the fraction of volatility assigned to BC- and high-frequency fluctuations is still too small. As the cost increases, the volatility of the RER and the fraction of volatility assigned to BC- and high-frequency fluctuations increase. A value of ι = 375 allows the model to get very close to matching the volatility of the RER and of output growth in the data and also improves the fit to the shape of the spectrum. Yet, too much weight is still placed on the low-frequency movements (82.5% of fluctuations at low frequencies in the model versus 72.2% in the data for $\iota =$ 375), i.e., the "excess persistence of the RER" puzzle is not fully solved. As *ι* grows, the model generates too much RER volatility but the share of variance assigned to low-frequency fluctuations remains higher than in the data. Hence, input adjustment costs can dramatically help to replicate RER volatility, even for large values of θ , but not to solve the puzzle completely. In what follows, we explain why input adjustment costs can help generate more RER volatility in the model. In the next section, we analyze how the interaction between input adjustment costs and a lower home bias can help in matching the spectrum of the RER.

In Fig. 3 we plot the IRFs to a home-country TFP shock for different values of ι to understand how this parameter shapes the behavior of the RER. When $\iota = 0$, standard results in the IRBC literature apply (see Backus et al. (1992)). When a TFP shock hits the home-country economy, we get the usual effect from an IRBC model: output, consumption, investment and hours worked increase in the home country, while in the foreign country, output, investment and hours worked decline, and consumption increases. As output expands, the demand for home- and foreign-country intermediate goods increases, although it increases more for home-country intermediate goods. In the foreign country, investment declines because foreign-country households buy home-country bonds to invest in the home country, with higher productivity, instead of foreign-country capital. Hours decline because of the associated decline of the marginal product of capital. Right away, foreign-country households increase their consumption because of an income effect related to future spillovers from the home-country technological improvement and higher returns on their bond holdings in the home country. In addition, this income effect leads the foreigncountry households to supply even less labor. As output decreases in the foreign country, the demand for home- and foreign-country intermediate goods also decreases.

As the literature has pointed out, the reaction of the RER is not too large but very persistent. The peak of the IRF happens after 20 quarters and the half-life is reached after more than 50 quarters. This highly persistent response of the RER is related to the "excess persistence of the RER" puzzle: regardless of the value of θ , far too much weight is placed at low-frequency movements. As a result of the decline in the price of home-country intermediate goods, and the increase in both the price and the quantity of foreign-country intermediate goods, a trade deficit for the home country emerges. This implies that variable D_t , which denotes the holding of bonds by the home-country household, becomes negative (see Eq. (8)). The variable D_t also denotes the net foreign asset position (NFA) of the home country. Thus, when a TFP shock hits the home country, its NFA position becomes negative in order to finance higher investment.

Introducing input adjustment costs leads to important changes in the behavior of some variables. The larger ι , the closer Y_H and Y_F need to move in order to avoid reducing the efficiency of the foreigncountry intermediate input. Without input adjustment costs Y_H increases more than Y_F , but the presence of the costs leads to a reduction in this difference. Something similar happens to Y_H^* and Y_F^* . As a result, the home-country demand for home-country intermediate goods increases less and the demand for foreign-country intermediate inputs increases more (when compared with the case of $\iota = 0$). This implies that, the larger the ι , the larger is the trade deficit that the home country runs (or the worse is its NFA position). This is key to inducing more RER volatility. Why is this the case? An inspection of the risk-sharing condition across countries gives us the answer. The linearized risk-sharing equation of the model reads as follows:

$$\widehat{rer}_{t} = E_{t} \Big[\widehat{rer}_{t+1} + \left(\hat{\lambda}_{t+1} - \hat{\lambda}_{t} \right) - \left(\hat{\lambda}_{t+1}^{*} - \hat{\lambda}_{t}^{*} \right) \Big] - \frac{\phi}{\beta} d_{t}$$

$$= E_{t} \sum_{i=0}^{\infty} \left\{ \Big[\left(\hat{\lambda}_{t+i+1} - \hat{\lambda}_{t+i} \right) - \left(\hat{\lambda}_{t+i+1}^{*} - \hat{\lambda}_{t+i}^{*} \right) \Big] - \frac{\phi}{\beta} d_{t+i} \right\}$$
(12)

where lower case variables with a hat (such as \hat{rer}_t) denote logdeviations from steady-state values and lower case variables (in this case, just d_t) denote deviations from steady-state values (this is the case because in the steady state, D = 0). Leaving aside changes in the relative marginal utilities of consumption, Eq. (12) links movements in the RER with the expected discounted sum of movements in the NFA position. Hence, the larger the input adjustment costs, the larger the NFA deterioration and the larger the depreciation of the RER. In fact, the NFA movements will mostly drive the behavior of the RER because households dislike changes in the marginal utility of consumption.

Therefore, there are two channels through which the introduction of input adjustment costs increases the volatility of the RER in the short run in the model. First, the input adjustment costs make relative quantities less sensitive to changes in relative prices, and this increases the volatility of the terms of trade and the RER. But at the same time, the volatility of net exports and net foreign assets increases, which feeds back into higher exchange rate volatility through Eq. (12). The large effects of input adjustment costs on RER fluctuations are important in the short run, when the costs play a role. In the long run, these adjustment costs dissipate and because of a large θ , RER fluctuations are dramatically reduced. Hence, the adjustment costs of imported inputs and the large long-run elasticity of substitution allow us to increase the size of RER fluctuations in the short run (because of large movements of the NFA in the short run due to the cost) and reduce them in the long run (because of a large θ). At this point, it is relevant to highlight that the feedback channel (between larger NFA volatility and larger RER short-run depreciation because of input adjustment costs) would not operate under complete markets. Hence, incomplete markets are a crucial part of the story.

¹⁰ This exercise emphasizes the importance of a low trade elasticity, at least in the short run. A low trade elasticity also helps in accounting for the failure of international risk sharing and other features of the data, at least at business cycle frequencies (see Corsetti et al. (2008b)).



Fig. 3. IRF to a home TFP shock when ι changes.

An alternative way to understand the mechanism is to analyze how the relationship between relative quantities of intermediate inputs and their relative prices changes across time once input adjustment costs are introduced. In Fig. 4, we compute a "pseudo-elasticity" of substitution when input adjustment costs are introduced as a function of time.



Fig. 4. Implied elasticity when *ι* changes.

In the baseline model, the elasticity of substitution between home and foreign goods is constant and equal to:

$$\frac{\partial \log(Y_{H,t}/Y_{F,t})}{\partial \log(P_{H,t}/P_{F,t})} = -\theta.$$

Computing the elasticities of substitution is not straightforward in the model with input adjustment costs (see Eqs. (10) and (11)). As a short cut, we compute the ratio:

$$\theta_k^{pseudo} = -\frac{\hat{y}_{H,t+k} - \hat{y}_{F,t+k}}{\hat{p}_{H,t+k} - \hat{p}_{F,t+k}}$$

at several time horizons k based on the IRFs to a home-country TFP shock presented in Fig. 4. The $\iota = 0$ case trivially delivers a constant elasticity of substitution of $\theta = 3$. The introduction of input adjustment costs delivers a short-term elasticity that is very low and close to zero (the limiting case of zero would be a Leontieff production function for the final good). Over time, the elasticity slowly increases to its long-run value of 3. Thus, introducing input adjustment costs allows us to have low short-run elasticities (that increase RER volatility at BC frequencies) with higher long-run elasticities (that lower RER volatility at lower frequencies). This mechanism goes a long way toward getting the shape of the spectrum right, but it does not fully solve the "excess persistence of the RER" puzzle.

6.2. The role of lower home bias

As we have shown in the previous subsection, there are limits to how much input adjustment costs help to solve the "excess persistence of the RER" puzzle. Here, we show how combining those costs with a lower home bias than the one used in the benchmark calibration helps solve the puzzle. In order to be able to compare our results with those in the existing literature, and in particular with Heathcote and Perri (2002), in the benchmark calibration we have chosen a value of $\omega = 0.9$. In the previous subsection, we have examined how far the model goes in explaining the data with cointegrated TFP processes and input adjustment costs. Unfortunately, it does not go far enough. As Heathcote and Perri (2002) emphasize, this value of ω is chosen to match the ratio of imports/output in the U.S. However, the imports/output ratio has been increasing over time, particularly in the last decade.

In order to see the evolution of this ratio, in Fig. 5 we plot the ratio of imports to GDP (in nominal and in real terms). We also plot the ratio of imports to private demand (consumption plus investment), which is a measure that is closer to our model, where there is no government spending. As Fig. 5 shows, all the ratios have been steadily increasing over the last four decades, from the single digits to values between 15 and 20% during the last ten years. More data are needed to assess and compute the steady-state imports–output ratio of the U.S. economy. But it is safe to assume that with increased globalization and trade liberalization worldwide, we can expect the ratio to be higher (and hence ω to be lower) than what has been typically calibrated in international macroeconomic models.

Hence, we study the implications of assuming a lower value of ω . In Table 5, we present the results of allowing the input adjustment cost parameter to vary, while setting $\theta = 3$ and $\omega = 0.8$, and keeping the rest of parameters of the model as in the benchmark calibration. Comparing Tables 4 and 5, we can see that absent input adjustment costs (i.e., $\iota = 0$), lowering the home bias only leads to lower RER volatility, as expected in an IRBC model. However, when ι increases, RER volatility increases, and the fraction of the volatility allocated to BC frequencies also increases, while the fraction of the volatility allocated to the lower frequencies declines. Interestingly, the interaction of (i) a large long-run elasticity of substitution, $\theta = 3$, (ii) a low home bias, and (iii)

The role of input adjustment costs with lower home bias.

| | Standard deviation | | Frequency of RER | | | Corr(C/C*, RER) | | |
|---------------|--------------------|---------------|------------------|------|------|-----------------|-------|-------|
| | RER | Output growth | Low | BC | High | Low | BC | High |
| U.S. data | 10.91 | 0.81 | 72.2 | 22.4 | 5.3 | -0.36 | -0.26 | -0.14 |
| $\iota = 0$ | 1.23 | 1.03 | 88.7 | 8.9 | 2.4 | 0.97 | 0.95 | 0.95 |
| $\iota = 100$ | 3.6 | 0.84 | 76.3 | 18.3 | 5.3 | 0.55 | 0.52 | 0.54 |
| $\iota = 200$ | 6.5 | 0.65 | 75.0 | 19.3 | 5.7 | 0.19 | 0.18 | 0.21 |
| $\iota = 325$ | 10.91 | 0.51 | 74.6 | 19.5 | 5.9 | -0.44 | -0.45 | -0.42 |

Note: See note in Table 2. The data for the correlation between the ratio of consumption and the real exchange rate at different frequencies come from Corsetti et al. (2012).

input adjustment costs allow the model to replicate both the standard deviation and the persistence of the RER (the area and the shape of the spectrum). In particular, when we set $\iota = 325$ to exactly match the volatility of the (log of the) RER, the model also explains the spectrum of the RER almost perfectly. We plot the spectrum of the data and the model in Fig. 6. The fit is remarkably good.

Next, we show why the combination of input adjustment costs and lower home bias helps the model replicate the shape of the spectrum. We compare the IRFs to a home TFP shock when $\theta = 3$ and $\iota = 325$, and the home bias parameter declines from $\omega = 0.9$ to $\omega = 0.8$, in Fig. 7. In the case of lower home bias, an increase in the home-country TFP shock is, on impact, more expansionary for the foreign country and less expansionary for the home country when compared with the higher home bias parameterization. When we consider $\omega = 0.8$, the foreign country imports larger amounts relative to the case of $\omega = 0.9$, which also raises its own output, consumption and investment more, while the opposite effect occurs in the home country. However, lower home bias has larger effects on foreign production than on foreign consumption, leading to a larger trade deficit and worsening of the net foreign asset position (more negative D_t) of the home country relative to the case of higher home bias. Hence, the initial response of the RER is to depreciate more relative to the case of $\omega = 0.9$. The mechanism behind this depreciation is, again, reflected in Eq. (12). As the effect of



Fig. 5. Evolution of 1- ω in the United States.



Fig. 6. Comparison between the model and the data.

the TFP shock is transmitted to the foreign economy through the cointegration process, the lower home bias implies a more rapid reversion of the just-described responses, which translates into a faster return of the RER to steady-state values. This faster return implies that a lower share of the fluctuations of the RER is going to be concentrated at lower frequencies when $\omega = 0.8$.

In addition, we want to remark on two additional results from the extended model. On the positive side, our model can explain an important fact in international macroeconomics. Since the seminal paper by Backus and Smith (1993), the literature has been preoccupied with matching the correlation between the ratio of the relative consumption of two countries (C/C^*) and the RER at BC frequencies. This correlation tends to be close to one in the standard model, even under cointegrated shocks, while it is negative in the data. Corsetti et al. (2008a) were the first ones to be able to explain this correlation at BC frequencies, with different specifications of international asset markets, persistence of productivity shocks, and the elasticity of substitution across goods. Recently, Corsetti et al. (2012) compute correlations at low, BC and high frequencies. In Table 5, we report the numbers computed by Corsetti et al. (2012) for the U.S. and a sample of industrialized countries. Their evidence shows that this correlation is negative at all frequencies, but perhaps more negative at low frequencies (-0.36) than at BC (-0.26) and high frequencies (-0.14). As Table 5 shows, our extended model is able to explain the negative correlation at all frequencies, while a model with no input adjustment costs implies a correlation very close to one at all frequencies. This nice result is linked to the fact that foreign consumption increases more than domestic consumption when the home bias is lower (Fig. 7). The mechanism is similar to that in Corsetti et al. (2008a), though these authors focus only on BC frequencies. That is, input adjustment costs lower the trade elasticity in the short run. Coupled with incomplete markets, the lower trade elasticity leads the correlation between (C/C^*) and the RER to be negative following shocks to the economy. The fact that foreign consumption rises more than home consumption following a home productivity shock in our extended model is very similar to the "positive international transmission" case pattern in Corsetti et al. (2008a). However, it



Fig. 7. IRF to a home TFP shock with lower home bias.

should be noted that this mechanism contradicts existing VAR evidence, where the opposite holds: domestic consumption increases more than foreign consumption, and the U.S. RER appreciates, rather than depreciate, as it does in Fig. 7.¹¹

On the negative side, Table 5 also shows that our model underpredicts the standard deviation of output growth, when compared with the results discussed in Table 4. In fact, Tables 4 and 5 show that the closer the model gets to explaining the RER spectrum, the worse it does in terms of explaining the standard deviation of U.S. real GDP growth. A natural question to ask, given the focus on spectral analysis in this paper, is how well does the model fit the spectrum of U.S. real GDP growth? The answer is not so well. In Table 6, we consider the cases of the Heathcote and Perri (2002) model with either nonstationary or stationary TFP shocks under different elasticities of substitution, and the preferred extended model in Table 5 that matches the spectrum of the RER with non-stationary TFP shocks, a high elasticity of substitution $\theta = 3$ input adjustment costs with $\iota = 325$, and lower home bias ($\omega = 0.8$). In the data, low-frequency fluctuations are not important (13.7%), but BC-frequency fluctuations are somewhat important (37.5%), and roughly half of the fluctuations are high-frequency. In all versions of the Heathcote and Perri (2002) model, most volatility is at high frequency (around 75% in all cases), while volatility at BC frequencies is not important (around 18.8%) and volatility at low frequencies is marginal (around 6%).¹² The preferred model does slightly better at matching the shape of the spectrum, but it does not get close to explaining the data. Therefore, the propagation mechanisms studied in this paper help explain the spectrum of the RER but not that of real GDP growth, which mainly inherits the properties of the exogenous TFP process. Addressing this issue would be an interesting avenue for future research. However, in order to match the spectrum for output growth, features would need to be introduced that may likely increase the importance of low-frequency movements in the RER. As a result, this would likely tend to exacerbate the "excess persistence of RER puzzle."

7. Concluding remarks

In this paper, we have shown that most of the volatility of the RER can be assigned to low frequencies (below BC frequencies). Therefore, it makes sense to ask if IRBC models can replicate the spectrum of the RER when no filter is applied to either the actual data or the simulated data coming from the model. Filtering the RER implies removing lowfrequency movements and eliminating most of the fluctuations of the RER. When matching the spectrum of the RER the challenge is twofold. First, we need to match its area (the volatility of the RER) and, second, its shape (the share of variance assigned to different frequencies). In Section 4, we have presented a standard version of a two-country, two-good IRBC model, in the spirit of Heathcote and Perri (2002), that includes cointegrated TFP shocks across countries as in Rabanal et al. (2011). This baseline model is capable of explaining the volatility of the RER (the area below the spectrum), but places too much weight on low-frequency movements (it cannot explain the shape of the spectrum). We call this shortcoming of the model the "excess persistence of the RER" puzzle.

In Section 5 we study whether modeling TFP shocks as stationary processes or adding IST shocks to our baseline model helps to solve the puzzle. We conclude that they do not. In Section 6 we try a new venue. We extend the baseline model to allow for adjustment costs in the use of intermediate inputs as in Erceg et al. (2006). We conclude

Table 6

Implications for output growth.

| | | Frequency | | |
|-----------------------------------|--------------------|-----------|------|------|
| | Standard deviation | Low | BC | High |
| U.S. data | 0.81 | 13.7 | 37.5 | 48.8 |
| Non-stationary, $\theta = 0.85$ | 0.75 | 6.4 | 18.8 | 74.8 |
| Non-stationary, $\theta = 062$ | 0.64 | 6.6 | 18.8 | 74.6 |
| Non-stationary, $\theta = 1.5$ | 0.85 | 6.2 | 18.7 | 75.1 |
| Stationary, $\theta = 0.85$ | 0.92 | 5.9 | 18.7 | 75.4 |
| Stationary, $\theta = 062$ | 0.87 | 6.1 | 18.7 | 75.2 |
| Stationary, $\theta = 1.5$ | 0.97 | 5.7 | 18.7 | 75.6 |
| Preferred model, last row Table 5 | 0.51 | 9.5 | 18.9 | 71.6 |

that what is needed to solve the puzzle while still explaining the volatility of the RER is the interaction of three ingredients: The first ingredient is a large steady-state elasticity of substitution ($\theta = 3$). The second ingredient is the introduction of adjustment costs in intermediate inputs, which help lower the implied elasticity of substitution in the short run and hence increase RER volatility at BC frequencies. Our preferred calibrated valued is $\iota = 325$, which means that a deviation in the ratio of inputs of 1% implies that output will be about 0.375% smaller. And the third ingredient is lower home bias, which is consistent with the pattern of increased trade openness of the U.S. and other economies over the last several decades. Moreover, the model can explain the evidence on the correlation between the RER and the ratio of relative consumption at BC and low frequencies, but not the spectrum of real GDP growth.

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¹¹ See Corsetti et al. (2014), Enders and Müller (2009) and Nam and Wang (2010) for VAR evidence. In IRBC models with tradable goods only, it is possible to replicate the VAR evidence with incomplete markets and a very low constant elasticity of substitution of 0.22, as in Enders and M üller (2009). Nam and Wang (2010) show that DSGE models with tradable goods only, and augmented with nominal and real frictions, cannot explain the behavior of relative prices after TFP shocks.

¹² Real GDP growth basically behaves as white noise.

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